On the problem of determination of spring stiffness parameters for spring-mesh models

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Abstract. On account of having real-time behavior and being easy to implement, spring meshes have been used for modeling deformable objects. Determining spring stiffness parameters for simulation of soft objects with high accuracy still remains a challenge. Allen Van Gelder derived an approximate formula for determining spring stiffness parameters based on strain analysis. Even though the experimental result showed the effectiveness, the method has not been investigated from a quantitative point of view. In this paper we propose a quantitative method for determining spring stiffness parameters. Moreover we propose a method to improve the accuracy by way of introducing torsional spring into the conventional spring mesh model.

Keywords. Mass-spring system, Finite element method, Stiffness matrix, Deformable model, Medical simulation

1. Introduction

Surgical simulators have been developed to create environments to help train physicians in learning skills of surgical operations at many research centers. In Surgical simulators, the real-time interactive models of behaviors of deformable objects play important roles. Two important requirement of simulations are high accuracy and real-time. Spring meshes [4] are especially useful for soft tissue modeling in surgical simulation due to their real-time behavior. A spring mesh is a system of vertices and edges, in which each vertex is a mass, each edge is a linear spring, and linear springs are connected at the vertices. Determining spring stiffness parameters for simulation of soft objects with high accuracy still remains a challenge. In [1], Allen Van Gelder showed that an exact simulation of the deformable behavior of elastic material by spring mesh is impossible. He also derived an approximate formula for determining spring stiffness parameters based on strain analysis. Even though the experimental result showed the effectiveness, his method has not been investigated from a quantitative point of view.

In another approach [2], the improvement of the accuracy of the spring mesh method is carried out via the determination of mesh topology. The main idea is to compare the deformation of behavior of a learning model with that of a reference model and to optimize the parameters of the learning model by way of using genetic algorithms. This approach is time-consuming and not effective for practical simulators. In this paper we propose a quantitative method for determining spring stiffness parameters. We also propose a method to improve the accuracy of conventional spring mesh models by way of introducing torsional springs into the conventional spring mesh models.

2. Methods

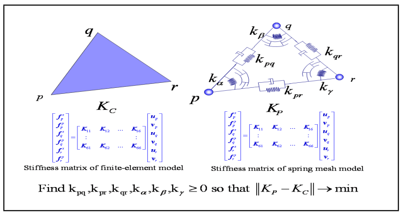

Figure 1. The proposed method for determination of spring stiffness parameters.

We propose a method for determination of spring stiffness parameters by way of minimization of the matrix norm of the stiffness matrix of a triangle mesh element of the spring mesh model with that of the referred finite-element model of the same isotropic elastic material. To improve the accuracy for simulation we introduce torsional springs into conventional spring mesh model. Figure 2 gives an illustration of the proposed method. In the finite-element formulation of a triangle element *pqr* the deformation at vertices are related to the external force loaded on the vertices through a stiffness matrix. Stiffness matrix is a $6 * 6$ matrix whose elements are expressions of the rest lengths of triangle's edges and two parameters of elasticity: the Young's modulus E and the Poisson coefficient ν. The stiffness matrix for the same triangle of springs is again a $6 * 6$ matrix whose elements are related only on the rest lengths, the rest angles and spring stiffness parameters. The method for constructing stiffness matrix of finite element model can be found in the literature of finite-element model [5]. The method for constructing stiffness matrix of torsional spring mesh model will be published in another paper. The matrix norm used in our method is Frobenious norm. It is obvious that the values of spring stiffness parameters that minimize the matrix norm will ensure the behavior of spring mesh model is similar to that of finite-element model.

3. Experimental Results

The relative error for evaluation of the proposed methods is as follows: $\varepsilon = \frac{\|K_p - K_c\|}{\|K_c\|}$, here K_p and K_c are stiffness matrices of spring mesh model and finite-element model. In comparison with the method of Allen Van Gelder, the following expression for calculating spring stiffness parameter of an edge is used: $k_c = \frac{E * area(T)}{c^2}$, where E, $area(T)$ and c are Young's modulus, area of the triangle and the length of the edge.

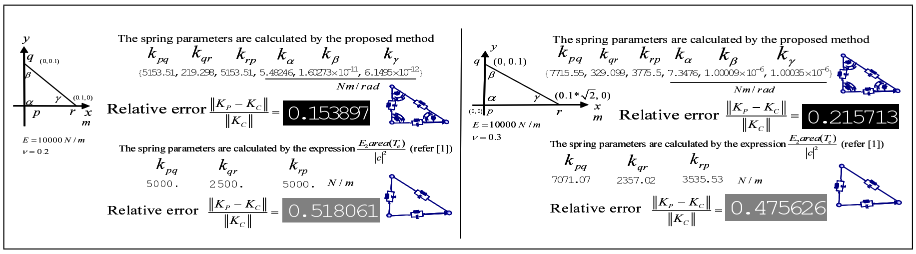

Figure 2. Experimental results on two different mesh elements

The experimental results show that in any case the relative error of the norm of the stiffness matrix of spring mesh model with torsional springs is always smaller than that of without torsional springs.

4. Conclusions

We proposed a quantitative method for determining spring stiffness parameters by way of minimizing stiffness matrix norm of spring mesh model and finite-element model, and a method to improve the accuracy of conventional spring mesh method by way of introducing torsional spring into the conventional spring mesh model. The experimental results show the effectiveness of the proposed method.

Acknowledgement

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